Calculus AB

5-3

Inverse Functions

Inverse of a function

Blue Collar Definition - Two functions are inverses if
they cancel each other out.

Graphical Definition - Two functions are inverses if
$(1,2) \to (2,1)$ their graphs are reflections about $y=x$. x & y are switched.

Mathematician's Definition - Two functions $f(x)$ and $g(x)$ are inverses iff
1) $f(g(x)) = x$
2) $g(f(x)) = x$

Show that f and g are inverse functions. (pg 349)

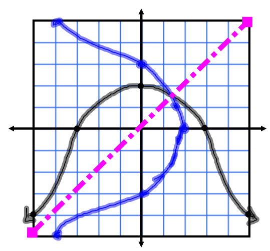
2) $f(x) = 3 - 4x$ $\qquad g(x) = \frac{3 - x}{4}$

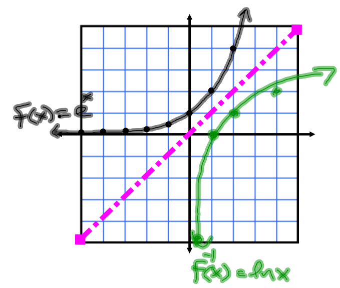
a) $f(g(x)) = 3 - 4\left(\frac{3-x}{4}\right)$
$= 3 - 3 + x$
$= x$ ✓

b) $g(f(x)) = \frac{3 - (3-4x)}{4} = \frac{3-3+4x}{4} = x$ ✓

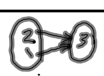
□

Sketch the inverse of each graph. Is the inverse a function?





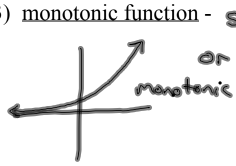
Definitions

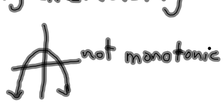


1) function - a rule or a map that assigns each input to exactly one output.

2) one-to-one function - Function where each output is assigned from exactly one input.

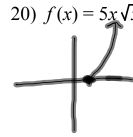
3) monotonic function - strictly increasing or strictly decreasing.





Use a graphing utility to graph the function. Determine whether it is one-to-one on its entire domain.

20) $f(x) = 5x\sqrt{x - 1}$



yes

Find the inverse function of f.

32) $f(x) = 3\sqrt[5]{2x - 1}$

$x = 3\sqrt[5]{2y-1}$

$\frac{x}{3} = \sqrt[5]{2y-1}$

$\left(\frac{x}{3}\right)^5 = 2y-1$

$\frac{\left(\frac{x}{3}\right)^5 + 1}{2} = y$

$f^{-1}(x) = \frac{\left(\frac{x}{3}\right)^5 + 1}{2}$

Use the derivative to determine whether the function is strictly monotonic on its entire domain and therefore has an inverse function.

44) $f(x) = (x+a)^3 + b$

$F'(x) = 3(x+a)^2(1)$

$0 = 3(x+a)^2$

$x = -a$ is a c.p.

$F''(x) = 6(x+a)$

$= 6(-a+a)$

$= 0$ p.o.i

yes, monotonic.

because the critical point is a point of inflection, The graph will be strictly increasing or decreasing. In this case, increasing.

Derivatives of Inverses

Given $f(x)$ and its inverse $f^{-1}(x)$, $f'(c) = \dfrac{1}{(f^{-1})'(c)}$

$m = \dfrac{\Delta y}{\Delta x}$

Translate the above definition into words:

the slope of the Function and its inverse are reciprocals

Let f be a function that is differentiable on an interval I.
If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$ and

$$g'(x) = \frac{1}{f'(g(x))} \quad \text{or} \quad (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Find $(f^{-1})'(a)$ for the function f and the real number a.

a is an input for the inverse

72) $f(x) = 5 - 2x^3$; $a = 7$

output for F(x)

$F'(x) = -6x^2$

$(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$7 = 5 - 2x^3$

$2 = -2x^3$

$-1 = x^3$

$-1 = x$

$F(-1) = 7$

$\boxed{F^{-1}(7) = -1}$

$(F^{-1})'(x) = \dfrac{1}{-6(F^{-1}(7))^2}$

$= \dfrac{1}{-6(-1)^2}$

$= -\dfrac{1}{6}$

Assignment:

Pg. 349
1 - 45 odd,
71 - 93 odd.